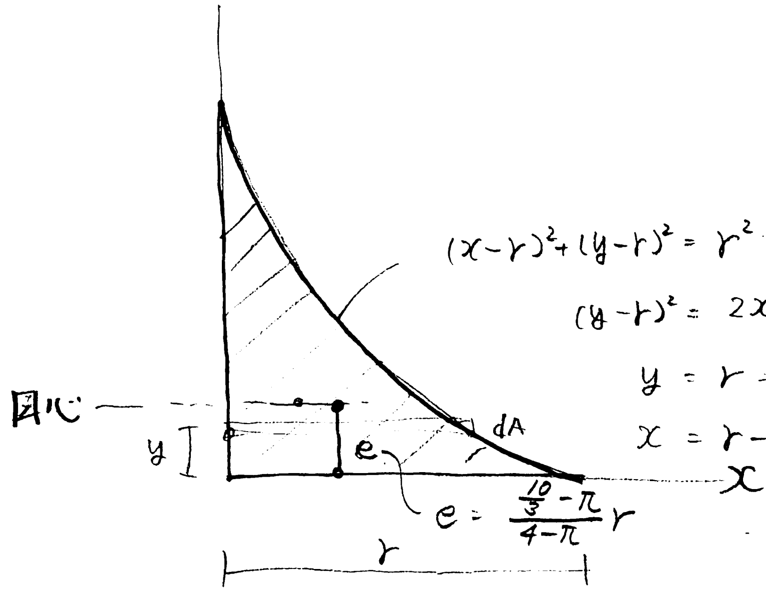



$(x-r)^2+(y-r)^2=r^2$

$(y-r)^2=2xr-x^2$

$y=r-\sqrt{2xr-x^2}$

$x=r-\sqrt{2yr-y^2}$

断面1次モーメント x軸まわり

$$S_x=\int y\,dA=\int y\cdot x\,dA=\int_0^r y\left(r-\sqrt{2yr-y^2}\right)dy$$

$$=r\left[\frac{y^2}{2}\right]_0^r-\int_0^r y\sqrt{r^2-(r-y)^2}\,d\theta$$

$$=\frac{r^3}{2}-I_1$$

$r-y=r\sin\theta$ とおく, $\theta=\frac{\pi}{2}\to\pi$

$$I_1=\int_{\frac{\pi}{2}}^{\pi}(r-r\sin\theta)\sqrt{r^2-r^2\sin\theta}\cdot(-r\cos\theta)\,d\theta$$

$$=r^3\int_{\frac{\pi}{2}}^{\pi}(1-\sin\theta)\cos^2\theta\,d\theta=r^3\int_{\frac{\pi}{2}}^{\pi}\left(\frac{\cos2\theta+1}{2}-\sin\theta\cos^2\theta\right)d\theta$$

$$=r^3\left[\frac{\sin2\theta}{4}+\frac{1}{2}\theta+\frac{\cos^3\theta}{3}\right]_{\frac{\pi}{2}}^{\pi}$$

$$=r^3\left(\frac{\pi}{2}-\frac{1}{3}-\left(\frac{\pi}{4}\right)\right)=r^3\left(\frac{\pi}{4}-\frac{1}{3}\right)$$

∴ $S_x=\left(\frac{5}{6}-\frac{\pi}{4}\right)r^3$

∴ 重心 $e=\frac{S_x}{A}=\frac{\left(\frac{5}{6}-\frac{\pi}{4}\right)r^3}{r^2-\frac{\pi r^2}{4}}=\boxed{\frac{\left(\frac{10}{3}-\pi\right)}{(4-\pi)}r}$

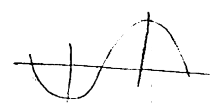
断面2次モーメント（x軸まわり）

$$I_x = \int y^2 dA = \int y^2 x\,dy = \int_0^r y^2\left(r-\sqrt{r^2-(r-y)^2}\right)dy$$

$$= r\left[\frac{y^3}{3}\right]_0^r - \int_0^r y^2\sqrt{r^2-(r-y)^2}\,dy$$

$$= \frac{r^4}{3} - I_2$$

$$I_2 = \int_0^r y^2\sqrt{r^2-(r-y)^2}\,dy$$



$r-y = r\sin\theta$ とおく。 $-\frac{dy}{d\theta} = r\cos\theta$, $\theta = \frac{\pi}{2} \to \pi$.

$$I_2 = \int_{\frac{\pi}{2}}^{\pi} (r-r\sin\theta)^2\, r\cos\theta\cdot(-r\cos\theta)\,d\theta$$

$$= r^4\int_{\frac{\pi}{2}}^{\pi} (1-\sin\theta)^2\cos^2\theta\,d\theta = r^4\int (1-2\sin\theta+\sin^2\theta)\cos^2\theta\,d\theta$$

$$= r^4\int\left(2\cos^2\theta - 2\cos^2\theta\sin\theta + (1-\cos^2\theta)\cos^2\theta\right)d\theta$$

$$= r^4\int_{\frac{\pi}{2}}^{\pi}\left(2\cdot\frac{\cos2\theta+1}{2} - 2\cos^2\theta\sin\theta - \cos^4\theta\right)d\theta$$

$$= r^4\left[\frac{\sin2\theta}{2}+\theta+\frac{2}{3}\cos^3\theta\right]_{\frac{\pi}{2}}^{\pi} - r^4\int_{\frac{\pi}{2}}^{\pi}\cos^4\theta\,d\theta$$

$$= r^4\left(\pi-\frac{2}{3}-\left(\frac{\pi}{2}\right)\right) - r^4\left\{\left[\frac{1}{4}\sin\theta\cos^3\theta\right]_{\frac{\pi}{2}}^{\pi} + \frac{3}{4}\int\cos^2\theta\,d\theta\right\}$$

$$= r^4\left(\frac{\pi}{2}-\frac{2}{3}\right) - r^4\cdot\frac{3}{4}\int_{\frac{\pi}{2}}^{\pi}\cos^2\theta\,d\theta \quad \left(\to \frac{\cos2\theta+1}{2}\right)$$

$$= r^4\left(\frac{\pi}{2}-\frac{2}{3}\right) - r^4\cdot\frac{3}{4}\left[\frac{\sin2\theta}{4}+\frac{\theta}{2}\right]_{\frac{\pi}{2}}^{\pi}$$

$$= r^4\left(\frac{\pi}{2}-\frac{2}{3}\right) - r^4\cdot\frac{3}{4}\left(\frac{\pi}{2}-\frac{\pi}{4}\right)$$

$$= r^4\left(\frac{5}{16}\pi-\frac{2}{3}\right)$$

$$\int_{\frac{\pi}{2}}^{\pi}\cos^4\theta\,d\theta = \int_0^{\frac{\pi}{2}}\sin^4\theta\,d\theta = \frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2} = \frac{3}{16}\pi$$

$$\therefore\ I_x = \frac{r^4}{3} - r^4\left(\frac{5}{16}\pi-\frac{2}{3}\right) = \boxed{r^4\left(1-\frac{5}{16}\pi\right)}$$

図心まわりの断面2次モーメント

$$= (I_x - Ae^2)$$

$$I_{x0} = r^4\left(1-\frac{5}{16}\pi\right) - \left(r^2 - \frac{\pi r^2}{4}\right)\cdot\left(\frac{\frac{10}{3}-\pi}{(4-\pi)}\right)^2 r^2$$

$$= r^4\left\{1-\frac{5}{16}\pi - \underbrace{\frac{1}{4}\frac{\left(\frac{10}{3}-\pi\right)^2}{4-\pi}}_{A}\right\}$$

$$A = -\frac{1}{4}\frac{\left(\frac{10}{3}-\pi\right)^2}{4-\pi} = -\frac{1}{4}\cdot\frac{\pi^2-\frac{20}{3}\pi+\frac{100}{9}}{4-\pi} = -\frac{1}{4}\frac{-\pi(4-\pi)-\frac{8}{3}\pi+\frac{100}{9}}{4-\pi}$$

$$= \frac{\pi}{4} + \frac{\frac{8}{3}\pi - \frac{100}{9}}{4(4-\pi)} = \frac{\pi}{4} + \frac{1}{4}\cdot\frac{-\frac{8}{3}(4-\pi)-\frac{4}{9}}{4-\pi} = \frac{\pi}{4} - \frac{2}{3} - \frac{1}{9}\frac{1}{4-\pi}$$

$$\therefore \underline{I_{x0}} = r^4\left(1-\frac{5}{16}\pi + \frac{\pi}{4} - \frac{2}{3} - \frac{1}{9}\frac{1}{4-\pi}\right)$$

$$= \boxed{r^4\left(\frac{1}{3} - \frac{\pi}{16} - \frac{1}{9}\frac{1}{4-\pi}\right)}$$