不規則波による応答.

$$f(t) = \int_0^t I'(t-\tau)\, g(\tau)\, d\tau$$

地動加速度.

$$\ddot{y} + 2h\omega\dot{y} + \omega^2 y = -g(t)$$

$$s^2 Y(s) - sy(0) - y'(0) + 2h\omega\left(sY(s) - y(0)\right) + \omega^2 Y(s) = \mathcal{L}\{-g(t)\}$$
$$= -G(s)$$

$y(0)=0,\quad y'(0)=0$ とすると.

$$(s^2 + 2h\omega s + \omega^2)Y(s) = -G(s)$$

$$\therefore \frac{Y(s)}{G(s)} = \frac{-1}{s^2 + 2h\omega s + \omega^2}$$ 伝達関数.

$$\frac{I(s)}{U(s)} = \frac{I(s)}{\left(\frac{1}{s}\right)} = \frac{-1}{s^2 + 2h\omega s + \omega^2}$$

$$I(s) = \frac{-1}{s(s^2 + 2h\omega s + \omega^2)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2h\omega s + \omega^2}$$

$$A + B = 0,\quad (2h\omega A + C) = 0,\quad A\omega^2 = -1.$$

$$\therefore A = -\frac{1}{\omega^2},\quad \frac{1}{B} = \frac{1}{\omega^2}\quad C = \frac{2h}{\omega}$$

$$I(s) = -\frac{1}{\omega^2}\cdot\frac{1}{s} + \frac{\frac{1}{\omega^2}s + \frac{2h}{\omega}}{(s + h\omega)^2 + (\omega\sqrt{1-h^2})^2}$$

$$= -\frac{1}{\omega^2}\cdot\frac{1}{s} + \frac{1}{\omega^2}\cdot\frac{s + h\omega + \frac{h}{\omega}}{(s + h\omega)^2 + (\omega\sqrt{1-h^2})^2}$$

インデンシャル応答.

$$I(t) = -\frac{1}{\omega^2} + \frac{1}{\omega^2}\cdot e^{-h\omega t}\left(\cos\omega\sqrt{1-h^2}\,t + \frac{h}{\sqrt{1-h^2}}\sin\omega\sqrt{1-h^2}\,t\right)$$

$$I'(t) = \frac{1}{\omega^2}(-h\omega)e^{-h\omega t}\left(\cos\omega\sqrt{1-h^2}\,t + \frac{h}{\sqrt{1-h^2}}\sin\omega\sqrt{1-h^2}\,t\right)$$
$$+ \frac{1}{\omega^2}e^{-h\omega t}\left(-\omega\sqrt{1-h^2}\sin\omega\sqrt{1-h^2}\,t + h\omega\cos\omega\sqrt{1-h^2}\,t\right)$$
$$= e^{-h\omega t}\left(\frac{-h^2\omega - \omega(1-h^2)}{\omega^2\sqrt{1-h^2}}\right)\sin\omega\sqrt{1-h^2}\,t = \frac{-1}{\omega\sqrt{1-h^2}}e^{-h\omega t}\sin\omega\sqrt{1-h^2}\,t$$

$$\therefore f(t) = \int_0^t I'(t-\tau)\, g(\tau)\, d\tau = -\frac{1}{\omega\sqrt{1-h^2}}\int_0^t e^{-h\omega(t-\tau)}\sin\left\{\omega\sqrt{1-h^2}\,(t-\tau)\right\} g(\tau)\, d\tau$$

$$I''(t) = \frac{h\omega}{\omega\sqrt{1-h^2}}e^{-h\omega t}\sin\omega\sqrt{1-h^2}\,t + \frac{-1}{\omega\sqrt{1-h^2}}e^{-h\omega t}\cdot\omega\sqrt{1-h^2}\cos\omega\sqrt{1-h^2}\,t$$
$$= e^{-h\omega t}\left\{\frac{h}{\sqrt{1-h^2}}\sin\omega\sqrt{1-h^2}\,t - \cos\omega\sqrt{1-h^2}\,t\right\}$$

$$I'''(t) = (-h\omega)e^{-h\omega t}\left\{\frac{h}{\sqrt{1-h^2}}\sin\omega\sqrt{1-h^2}\,t - \cos\omega\sqrt{1-h^2}\,t\right\}$$
$$+ e^{-h\omega t}\left\{h\omega\cos\omega\sqrt{1-h^2}\,t + \omega\sqrt{1-h^2}\sin\omega\sqrt{1-h^2}\,t\right\}$$
$$= e^{-h\omega t}\left\{\frac{\omega(1-2h^2)}{\sqrt{1-h^2}}\sin\omega\sqrt{1-h^2}\,t + 2h\omega\cos\omega\sqrt{1-h^2}\,t\right\}$$

相対速度応答

$$f'(t) = \int_0^t I''(t-\tau)\, g(\tau)\, d\tau$$

$$= \int_0^t e^{-h\omega(t-\tau)} \left\{ \frac{h}{\sqrt{1-h^2}} \sin\left\{\omega\sqrt{1-h^2}(t-\tau)\right\} - \cos\left\{\omega\sqrt{1-h^2}(t-\tau)\right\} \right\} g(\tau)\, d\tau$$

絶対加速度応答

$$f''(t) = \int_0^t I'''(t-\tau)\, g(\tau)\, d\tau$$

$$= \int_0^t e^{-h\omega(t-\tau)} \left[\frac{\omega(1-2h^2)}{\sqrt{1-h^2}} \sin\left\{\omega\sqrt{1-h^2}(t-\tau)\right\} + 2h\omega \cos\left\{\omega\sqrt{1-h^2}(t-\tau)\right\} \right] g(\tau)\, d\tau$$