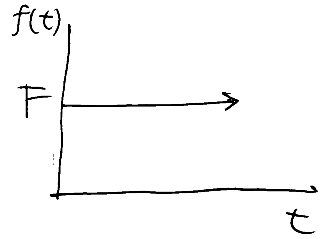
○ステップ外力に対する応答.

$$m\ddot{x} + c\dot{x} + kx = F$$

$$\ddot{x} + 2h\omega\dot{x} + \omega^2 x = \frac{F}{m}$$



$$y = y_0 + Y$$

$$Y = e^{-h\omega t}\left(c_1 \cos \omega\sqrt{1-h^2}\,t + c_2 \sin \omega\sqrt{1-h^2}\,t\right)$$

$$y_0 = \frac{1}{D^2 + 2h\omega D + \omega^2}\frac{F}{m}$$

$$= \frac{1}{\omega^2\{1-(-\frac{D^2}{\omega^2} - \frac{2h}{\omega}D)\}}\frac{F}{m}$$

$$= \frac{1}{\omega^2}\cdot\frac{F}{m}$$

$$\therefore y = \frac{1}{\omega^2}\frac{F}{m} + e^{-h\omega t}\left(c_1 \cos\omega\sqrt{1-h^2}\,t + c_2 \sin\omega\sqrt{1-h^2}\,t\right)$$

初期条件. $t=0$ で $y=0$, $\dot{y}=0$ とすると.

$$y(t=0) = \frac{1}{\omega^2}\frac{F}{m} + c_1 = 0 \qquad \therefore c_1 = -\frac{1}{\omega^2}\frac{F}{m}$$

$$\dot{y}(t=0) = -h\omega e^{-h\omega t}\left(c_1\cos\omega\sqrt{1-h^2}\,t + c_2 \sin\omega\sqrt{1-h^2}\,t\right)$$

$$+ e^{-h\omega t}\left(-c_1\omega\sqrt{1-h^2}\sin\omega\sqrt{1-h^2}\,t + c_2\omega\sqrt{1-h^2}\cos\omega\sqrt{1-h^2}\,t\right)$$

$t=0$

$$= -h\omega c_1 + c_2\omega\sqrt{1-h^2} = 0$$

$$\therefore c_2 = \frac{h}{\sqrt{1-h^2}}c_1$$

$$\therefore y = \frac{1}{\omega^2}\frac{F}{m} + e^{-h\omega t}\left(-\frac{1}{\omega^2}\frac{F}{m}\cos\omega\sqrt{1-h^2}\,t - \frac{1}{\omega^2}\frac{F}{m}\frac{h}{\sqrt{1-h^2}}\sin\omega\sqrt{1-h^2}\,t\right)$$

$$= \frac{1}{\omega^2}\frac{F}{m}\left\{1 - e^{-h\omega t}\left(\cos\omega\sqrt{1-h^2}\,t + \frac{h}{\sqrt{1-h^2}}\sin\omega\sqrt{1-h^2}\,t\right)\right\}$$

$$= \frac{1}{(\sqrt{\frac{k}{m}})^2}\frac{F}{m}\{\ \cdots\ \}$$

$$= \frac{F}{k}\left\{1 - e^{-h\omega t}\left(\cos\omega\sqrt{1-h^2}\,t + \frac{h}{\sqrt{1-h^2}}\sin\omega\sqrt{1-h^2}\,t\right)\right\}$$

$$\ddot{y} + 2h\omega\dot{y} + \omega^2 y = \frac{F}{m}$$

$$s^2F(s) - sy(0) - y'(0) + 2h\omega(sF(s) - y(0)) + \omega^2F(s) = \frac{F}{m}$$

$$F(s) = \underbrace{\frac{s}{s^2+2h\omega s+\omega^2}y(0) + \frac{y'(0)+2h\omega y(0)}{s^2+2h\omega s+\omega^2}}_{y_1(t)} + \underbrace{\frac{\frac{F}{m}}{s(s^2+2h\omega+\omega^2)}}_{y_0}$$

$$\therefore F_1(s) = \frac{s+h\omega}{(s+h\omega)^2+\omega^2(1-h^2)}y(0) + \frac{\omega\sqrt{1-h^2}}{(s+h\omega)^2+\omega^2(1-h^2)}\cdot\frac{y'(0)+h\omega y(0)}{\omega\sqrt{1-h^2}}$$

$$y_1(t) = e^{-h\omega t}\left(y(0)\cos\omega\sqrt{1-h^2}t + \frac{y'(0)+h\omega y(0)}{\omega\sqrt{1-h^2}}\sin\omega\sqrt{1-h^2}\right)$$

$$F_2(s) = \frac{\frac{F}{m}}{s(s^2+2h\omega s+\omega^2)} = \frac{F}{m}\cdot\left(\frac{A}{s}+\frac{Bs+C}{s^2+2h\omega s+\omega^2}\right)$$

$$1 = A(s^2+2h\omega s+\omega^2)+(Bs+C)s = (A+B)s^2+(2h\omega A+C)s+A\omega^2$$

$$\therefore A = \frac{1}{\omega^2}, \quad A+B=0, \quad B=-\frac{1}{\omega^2}, \quad C=-2h\omega A = -\frac{2h}{\omega}$$

$$\therefore F_2(s) = \frac{F}{m}\left(\frac{1}{\omega^2}\cdot\frac{1}{s} + \frac{-\frac{1}{\omega^2}s}{(s+h\omega)^2+\omega^2(1-h^2)} + \frac{-\frac{2h}{\omega}}{(s+h\omega)^2+\omega^2(1-h^2)}\right)$$

$$= \frac{F}{m}\left(\frac{1}{\omega^2}\cdot\frac{1}{s} + \frac{-\frac{1}{\omega^2}(s+h\omega)}{(s+h\omega)^2+\omega^2(1-h^2)} + \frac{\omega\sqrt{1-h^2}}{(s+h\omega)^2+\omega^2(1-h^2)}\cdot\frac{-\frac{h}{\omega}}{\omega\sqrt{1-h^2}}\right)$$

$$y_2(t) = \frac{F}{m}\left\{\frac{1}{\omega^2} + e^{-h\omega t}\left(-\frac{1}{\omega^2}\cos\omega\sqrt{1-h^2}t - \frac{h}{\omega^2\sqrt{1-h^2}}\sin\omega\sqrt{1-h^2}t\right)\right\}$$

$$= \frac{F}{m}\cdot\frac{1}{\omega^2}\left\{1 - e^{-h\omega t}(\cos\omega\sqrt{1-h^2}t + \frac{h}{\sqrt{1-h^2}}\sin\omega\sqrt{1-h^2}t\right\}$$

$\therefore$ $y(t) = y_1(t) + y_2(t)$.

$y(0) = y'(0) = 0$ のとき.

$y(t) = y_2(t)$.