・ 減衰自由振動.

$$m\ddot{x} + c\dot{x} + kx = 0.$$

$$\ddot{x} + 2h\omega\dot{x} + \omega^2 x = 0.$$

$$\frac{c}{m} = 2h\omega, \quad \frac{k}{m} = \omega^2$$

$x = e^{\lambda t}$ とおく.

$$\lambda^2 + 2h\omega\lambda + \omega^2 = 0.$$

$$\lambda = -h\omega \pm i\omega\sqrt{1-h^2}.$$

$$x(t) = C_1 e^{(-h\omega + i\omega\sqrt{1-h^2})t} + C_2 e^{(-h\omega - i\omega\sqrt{1-h^2})t}$$

$$= e^{-h\omega t}\left\{C_1 e^{i\omega\sqrt{1-h^2}t} + C_2 e^{-i\omega\sqrt{1-h^2}t}\right\}$$

$$= e^{-h\omega t}\left\{C_1\left(\cos\omega\sqrt{1-h^2}t + i\sin\omega\sqrt{1-h^2}t\right) + C_2\left(\cos\omega\sqrt{1-h^2}t - i\sin\omega\sqrt{1-h^2}t\right)\right\}$$

$$= e^{-h\omega t}\left\{(C_1 + C_2)\cos\omega\sqrt{1-h^2}t + i(C_1 - C_2)\sin\omega\sqrt{1-h^2}t\right\}$$

$$= e^{-h\omega t}\left(C_3\cos\omega\sqrt{1-h^2}t + C_4\sin\omega\sqrt{1-h^2}t\right)$$

$x(0) = a$, $\dot{x}(0) = v_0$ aとき.

$$x(0) = e^0(C_3\cdot 1 + C_4\cdot 0), \quad C_3 = x(0) = a.$$

$$\dot{x}(0) = -h\omega e^{-h\omega t}\left(C_3\cos\omega\sqrt{1-h^2}t + C_4\sin\omega\sqrt{1-h^2}t\right) + e^{-h\omega t}\left(C_3(-\omega\sqrt{1-h^2})\sin\omega\sqrt{1-h^2}t + C_4\cdot\omega\sqrt{1-h^2}\cos\omega\sqrt{1-h^2}t\right)$$

$x = 0$

$$= -h\omega\cdot C_3 + C_4\cdot\omega\sqrt{1-h^2} = v_0$$

$$C_4 = \frac{h\omega\cdot C_3}{\omega\sqrt{1-h^2}} + \frac{v_0}{\omega\sqrt{1-h^2}} = \frac{v_0 + h\omega a}{\omega\sqrt{1-h^2}}$$

$$\therefore \; y = e^{-h\omega t}\left(a\cdot\cos\omega\sqrt{1-h^2}t + \frac{v_0 + h\omega a}{\omega\sqrt{1-h^2}}\cdot\sin\omega\sqrt{1-h^2}t\right)$$

または.

$$y'' + 2h\omega y' + \omega^2 y = 0$$

$$s^2F(s) - sy(0) - y'(0) + 2h\omega(sF(s) - y(0)) + \omega^2F(s) = 0$$

$$F(s) = \frac{s}{s^2 + 2h\omega s + \omega^2} y(0) + \frac{y'(0) + 2h\omega y(0)}{s^2 + 2h\omega s + \omega^2}$$

$$= \frac{(s + h\omega)}{(s+h\omega)^2 + \omega^2(1-h^2)} y(0) + \frac{\omega\sqrt{1-h^2}}{(s+h\omega)^2 + \omega^2(1-h^2)} \cdot \frac{y'(0) + h\omega y(0)}{\omega\sqrt{1-h^2}}$$

$$\therefore\ y = y(0)e^{-h\omega t}\cos\omega\sqrt{1-h^2}\,t + \frac{y'(0) + h\omega y(0)}{\omega\sqrt{1-h^2}} \cdot e^{-h\omega t}\sin\omega\sqrt{1-h^2}\,t.$$