○ 強制外力に対する定常応答.

$$m\ddot{x}+c\dot{x}+kx = F\cos pt \quad \leftarrow \text{調和外力}$$

$$\ddot{x}+2h\omega\dot{x}+\omega^2 x = \frac{F}{m}\cos pt$$

$$y = \underbrace{y_0}_{\text{特解}} + \underbrace{Y}_{\text{余関数}}$$

$$Y = e^{-h\omega t}\left(C_1\cos\omega\sqrt{1-h^2}\,t + C_2\sin\omega\sqrt{1-h^2}\,t\right)$$

$$y_0 = \frac{1}{P(D)}F\cos pt = \frac{F}{m}\cdot\frac{1}{P(-p^2)}\cos pt$$

$$= \frac{F}{m}\cdot\frac{1}{D^2+2h\omega D+\omega^2}\cos pt$$

$$= \frac{F}{m}\cdot\frac{1}{(-p^2)+2h\omega D+\omega^2}\cos pt$$

$$= \frac{F}{m}\cdot\frac{(\omega^2-p^2)-2h\omega D}{(\omega^2-p^2)^2-(2h\omega D)^2}\cos pt$$

$$= \frac{F}{m}\cdot\frac{(\omega^2-p^2)-2h\omega D}{(\omega^2-p^2)^2-(2h\omega)^2\cdot D^2}\cos pt$$

$$= \frac{F}{m}\cdot\frac{(\omega^2-p^2)-2h\omega D}{(\omega^2-p^2)^2-(2h\omega)^2\cdot(-p^2)}\cos pt$$

$$= \frac{F}{m}\cdot\frac{(\omega^2-p^2)-2h\omega D}{(\omega^2-p^2)^2+(2h\omega p)^2}\cos pt$$

$$= \frac{F}{m}\cdot\frac{1}{(\omega^2-p^2)^2+(2h\omega p)^2}\left\{(\omega^2-p^2)\cos pt-2h\omega\cdot(-p)\sin pt\right\}$$

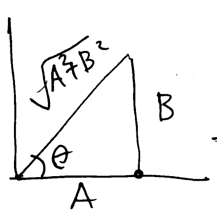
$$= \frac{F}{m}\cdot\frac{1}{(\omega^2-p^2)^2+(2h\omega p)^2}\left\{(\omega^2-p^2)\cos pt+2h\omega p\sin pt\right\}$$

$$= \frac{F}{m}\cdot\frac{\sqrt{(\omega^2-p^2)^2+(2h\omega p)^2}}{(\omega^2-p^2)^2+(2h\omega p)^2}\cdot\cos(pt-\theta)$$

$$= \frac{F/m}{\sqrt{(\omega^2-p^2)^2+(2h\omega p)^2}}\cos(pt-\theta)$$

$$\theta = \tan^{-1}\left(\frac{2h\omega p}{\omega^2-p^2}\right)$$

$$A\cos x+B\sin x = \sqrt{A^2+B^2}\cos(x-\theta)$$



$$\tan\theta = \frac{B}{A}, \quad \theta = \tan^{-1}\frac{B}{A}$$

$$\cos\theta = \frac{A}{\sqrt{A^2+B^2}}, \quad \sin\theta = \frac{B}{\sqrt{A^2+B^2}}$$

$$\sqrt{A^2+B^2}\cos(x-\theta)$$

$$= \sqrt{A^2+B^2}\left(\cos x\cos\theta+\sin x\sin\theta\right)$$

$$= \sqrt{A^2+B^2}\cdot\left(\frac{A}{\sqrt{A^2+B^2}}\cos x+\frac{B}{\sqrt{A^2+B^2}}\sin x\right) = A\cos x+B\sin x.$$