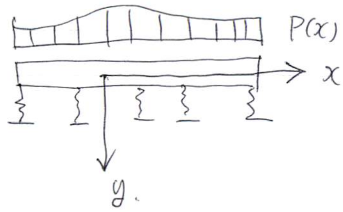
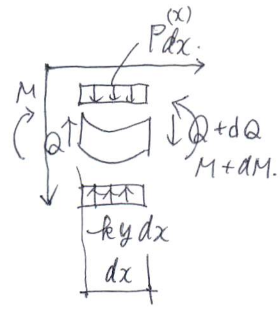
弾性床上の梁





○ 鉛直方向の釣り合い

$dQ + P(x)dx - ky\,dx = 0$

$\therefore \frac{dQ}{dx} = ky - P(x)$

○ モーメントの釣り合い

$Q\,dx = dM$

$\therefore Q = \frac{dM}{dx}$

○ $\frac{d^2M}{dx^2} = ky - P(x)$

$M = -EI\frac{d^2y}{dx^2}$ より

$-EI\frac{d^4y}{dx^4} = ky - P(x)$

弾性床上の梁

$\therefore \frac{d^4y}{dx^4} + \frac{k}{EI}y = \frac{P(x)}{EI}$

$\beta = \sqrt[4]{\frac{k}{4EI}}$ とおいて、

$\frac{d^4y}{dx^4} + 4\beta^4 = \frac{P(x)}{EI}$

$\beta = \sqrt[4]{\frac{k}{EI}}$ とすると
$\lambda^4 + \beta^4 = 0$.
$r^4e^{i4\theta} = \beta^4(\cos\pi + i\sin\pi)$
$r = \beta,\ 4\theta = \pi + 2n\pi$.
$\lambda = \beta e^{i\frac{\pi}{4}} = \beta(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = \frac{\beta}{\sqrt{2}} + \frac{\beta}{\sqrt{2}}i$
となってすっきりしない。

$y = y_p + y_h$
特解　一般解.

一般解を求める。

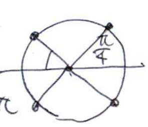
$y = e^{\lambda x}$ とおく。

$e^{\lambda x}(\lambda^4 + 4\beta^4) = 0$.

$\lambda = re^{i\theta}$ とおく。 $r^4e^{i4\theta} = -4\beta^4 = 4\beta^4(\cos\pi + i\sin\pi)$

$r = \sqrt{2}\beta$, $4\theta = \pi + 2n\pi$

$\theta = \frac{\pi}{4}, \frac{3}{4}\pi, \frac{5}{4}\pi, \frac{7}{4}\pi$



$\therefore \lambda = \sqrt{2}\beta(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}) = \beta(1+i)$
$= \beta(-1+i)$
$= \beta(-1-i)$
$= \beta(1-i)$

一般解.

$\therefore y = C_1e^{\beta(1+i)x} + C_2e^{\beta(-1+i)x} + C_3e^{\beta(-1-i)x} + C_4e^{\beta(1-i)x}$

$= C_1e^{\beta x}(\cos\beta x + i\sin\beta x) + C_2e^{-\beta x}(\cos\beta x + i\sin\beta x)$
$+ C_3e^{-\beta x}(\cos\beta x - i\sin\beta x) + C_4e^{\beta x}(\cos\beta x - i\sin\beta x)$.

$e^{\beta x}(\cos\beta x + i\sin\beta x) + e^{\beta x}(\cos\beta x - i\sin\beta x)$
$= 2e^{\beta x}\cos\beta x \to e^{\beta x}\cos\beta x$ も解

$-ie^{\beta x}(\cos\beta x + i\sin\beta x) + ie^{\beta x}(\cos\beta x - i\sin\beta x)$
$= 2e^{\beta x}\sin\beta x \to e^{\beta x}\sin\beta x$ も解.

$\therefore y = e^{\beta x}(C_1\cos\beta x + C_2\sin\beta x)$
$+ e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x)$
と cos, sin で表した場合.

または

$\cosh x = \frac{e^x + e^{-x}}{2}$　$\sinh x = \frac{e^x - e^{-x}}{2}$

$e^{\beta x}\cos\beta x + e^{-\beta x}\cos\beta x = (e^{\beta x} + e^{-\beta x})\cos\beta x$
$= 2\cosh\beta x\cos\beta x \to \cosh\beta x\cos\beta x$ も解.

$e^{\beta x}\sin\beta x + e^{-\beta x}\sin\beta x = (e^{\beta x} + e^{-\beta x})\sin\beta x$
$= 2\cosh\beta x\sin\beta x \to \cosh\beta x\sin\beta x$ も解.

$e^{\beta x}\cos\beta x - e^{-\beta x}\cos\beta x = (e^{\beta x} - e^{-\beta x})\cos\beta x$
$= 2\sinh\beta x\cos\beta x \therefore \sinh\beta x\cos\beta x$ も解.

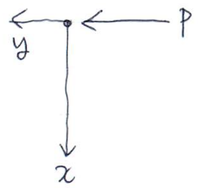
よって

$\therefore y = C_1\cosh\beta x\cos\beta x + C_2\cosh\beta x\sin\beta x$
$+ C_3\sinh\beta x\cos\beta x + C_4\sinh\beta x\sin\beta x$

No.

Date

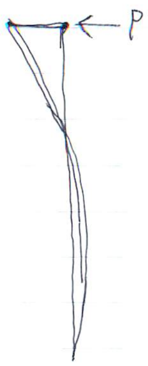
○ 杭の計算　杭頭固定.



$$y = e^{\beta x}(C_1\cos\beta x + C_2\sin\beta x) + e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x)$$

$x \to \infty$ で $y = 0$ とすると, $C_1 = C_2 = 0$.

$$\therefore y = e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x)$$



杭頭固定より.

$$\theta = \left.\frac{dy}{dx}\right|_{x=0} = 0. \qquad Q_{x=0} = -P.$$

$$\frac{d^2y}{dx^2} = -\frac{M(x)}{EI} \qquad \frac{d^3y}{dx^3} = -\frac{1}{EI}\cdot\frac{dM}{dx} = -\frac{1}{EI}Q \qquad \therefore Q = -EI\frac{d^3y}{dx^3}$$

$$-\beta e^{-\beta x}(C_3\cos\beta x + \cancel{C_4}\, C_3\sin\beta x)$$
$$-\beta e^{-\beta x}(C_3\sin\beta x - C_3\beta\cos\beta x)$$
$$= -2\beta e^{-\beta x} C_3(\cancel{\cos\beta x} + \sin\beta x)$$
$$= \frac{-P}{2EI\beta^2}e^{-\beta x}(\cos\beta x + \sin\beta x)$$

$$y' = -\beta e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x) + e^{-\beta x}(-C_3\beta\,\overset{\sin}{\cancel{\cos}}\beta x + C_4\beta\cos\beta x)$$

$$y'' = \beta^2 e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x) - \beta e^{-\beta x}(-C_3\beta\sin\beta x + C_4\beta\cos\beta x)$$
$$+ (-\beta)e^{-\beta x}(-C_3\beta\sin\beta x + C_4\beta\cos\beta x) + e^{-\beta x}(-C_3\beta^2\cos\beta x - C_4\beta^2\sin\beta x)$$

$$y'_{x=0} = -\beta C_3 + C_4\beta = 0. \quad \therefore C_3 = C_4.$$

$$\cancel{y''_{x=0} = \beta^2 C_3 - \beta^2 C_4 - \beta^2 C_4 - C_3\beta^2 = \frac{1}{(-EI)}\cdot(-P)}$$

$$\cancel{\therefore C_4 = \frac{1}{-2\beta^2}\cdot\frac{-P}{(-EI)} = -\frac{P}{2\beta^2 EI}}$$

$$\cancel{\therefore y = e^{-\beta x}\left(\frac{P}{2\beta^2 EI}\right)(+\cos\beta x - \sin\beta x)}$$

$$y''' = -\beta^3 e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x) + \beta^2 e^{-\beta x}(-C_3\beta\sin\beta x + \beta C_4\cos\beta x)$$
$$+ \beta^2 e^{-\beta x}(-C_3\beta\sin\beta x + C_4\beta\cos\beta x) - \beta e^{-\beta x}(-C_3\beta^2\cos\beta x - C_4\beta^2\sin\beta x)$$
$$+ \beta^2 e^{-\beta x}(-C_3\beta\sin\beta x + C_4\beta\cos\beta x) - \beta e^{-\beta x}(-C_3\beta^2\cos\beta x - C_4\beta^2\sin\beta x)$$
$$+ (-\beta)e^{-\beta x}(-C_3\beta^2\cos\beta x - C_4\beta^3\sin\beta x) + e^{-\beta x}(+C_3\beta^3\sin\beta x - C_4\beta^3\cos\beta x)$$

$$y'''_{x=0} = -\beta^3 C_3 + C_4\beta^3 + C_4\beta^3 + \beta^3 C_3 + C_4\beta^3 + C_3\beta^3 + C_3\beta^3 - C_4\beta^3$$
$$2C_3\beta^3 + 2C_4\beta^3 = 4C_3\beta^3 = -\frac{-P}{EI} = \frac{P}{EI}$$

$$\therefore C_3 = \frac{P}{4EI\beta^3}$$

$$y = e^{-\beta x}\left(\frac{P}{4EI\beta^3}\right)(\cos\beta x + \sin\beta x).$$

$$y_{x=0} = \frac{P}{4EI\beta^3}$$

$$y'' = \beta^2 e^{-\beta x}(C_3\cos\beta x + C_3\sin\beta x) - 2\beta e^{-\beta x}(-C_3\beta\sin\beta x + C_3\beta\cos\beta x)$$
$$+ e^{-\beta x}(-C_3\beta^2\cos\beta x - C_3\beta^2\sin\beta x)$$
$$= -2\beta^2 e^{-\beta x}(C_3\cos\beta x - C_3\sin\beta x) = \frac{P}{2EI\beta}e^{-\beta x}(-\cos\beta x - \sin\beta x)$$

$$M_{(x=0)} = -EIy''_{x=0} = -EI\cdot\left(-\frac{P}{2EI\beta}\right) = \frac{P}{2\beta}$$

$$-2\beta e^{-\beta x}\cdot\frac{P}{4EI\beta^3}\sin\beta x$$

$$\therefore y' = \frac{-P}{2EI\beta^3}\cancel{e^{-\beta x}}(\cos\beta x +$$
$$= \frac{-P}{2EI\beta^2}e^{-\beta x}\sin\beta x.$$

$$y'' = \frac{-P}{2EI\beta^2}\{-\beta e^{-\beta x}\sin\beta x + e^{-\beta x}\beta\cos\beta x\}$$
$$= \frac{-P}{2EI\beta}e^{-\beta x}(\cos\beta x - \sin\beta x)$$

$$y''' = \frac{-P}{2EI\beta}\{(-\beta)e^{-\beta x}(\cos\beta x - \sin\beta x) + e^{-\beta x}(-\beta\sin\beta x - \beta\cos\beta x)\}$$
$$= \frac{-P}{2EI\beta}\cancel{\beta}e^{-\beta x}(-\cos\beta x + \sin\beta x - \sin\beta x - \cos\beta x)$$
$$= \frac{-P}{2EI}e^{-\beta x}(-2\cos\beta x)$$
$$= \frac{P}{EI}e^{-\beta x}\cos\beta x.$$

$$\therefore M = -EIy'' = \frac{P}{2\beta}e^{-\beta x}(\cos\beta x - \sin\beta x)$$

$$Q = -EIy''' = \frac{-P}{1}e^{-\beta x}\cos\beta x.$$

まとめ.

$$y = \frac{P}{4EI\beta^3} e^{-\beta x}(\cos\beta x + \sin\beta x)$$

$$y' = \frac{-P}{2EI\beta^2} e^{-\beta x} \sin\beta x.$$

$$y'' = \frac{-P}{2EI\beta} e^{-\beta x}(\cos\beta x - \sin\beta x)$$

$$y''' = \frac{P}{EI} e^{-\beta x} \cos\beta x.$$

$$M = -EIy'' = \frac{P}{2\beta} e^{-\beta x}(\cos\beta x - \sin\beta x)$$

$$Q = -EIy''' = -P e^{-\beta x} \cos\beta x.$$

$$y_{x=0} = \frac{P}{4EI\beta^3}$$

$$M_{x=0} = \frac{P}{2\beta}$$

$$Q_{x=0} = -P$$

地中の最大曲りモーメント. 位置. $\beta x = \frac{\pi}{2}$ ∴ $x = \frac{\pi}{2}/\beta = 1.57\beta$.

$$M = \frac{P}{2\beta} \cdot e^{-\frac{\pi}{2}}\left(\cos\frac{\pi}{2} - \sin\frac{\pi}{2}\right) = -\frac{P}{2\beta} \cdot e^{-\frac{\pi}{2}} = -0.208 \cdot \frac{P}{2\beta}$$

No.

Date

e 杭頭ピン.

$$y = e^{\beta x}(C_1\cos\beta x + C_2\sin\beta x) + e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x)$$

$$= e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x)$$

$x\to\infty$ $y=0$ より. $C_1 = C_2 = 0$

杭頭ピンより.

$y''_{x=0} = 0$, $Q(x) = -P$

$$y' = -\beta e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x) + e^{-\beta x}(-\beta C_3\sin\beta x + \beta C_4\cos\beta x)$$

$$= -\beta e^{-\beta x}(C_3\cos\beta x + C_4\sin\beta x + C_3\sin\beta x - C_4\cos\beta x)$$

$$= -\beta e^{-\beta x}\{(C_3 - C_4)\cos\beta x + (C_3 + C_4)\sin\beta x\}$$

$$y'' = \beta^2 e^{-\beta x}\{(C_3 - C_4)\cos\beta x + (C_3 + C_4)\sin\beta x\} + (-\beta)e^{-\beta x}\{-\beta(C_3 - C_4)\sin\beta x + (C_3 + C_4)\beta\cos\beta x\}$$

$$= \beta^2 e^{-\beta x}\{(C_3 - C_4)\cos\beta x + (C_3 + C_4)\sin\beta x + (C_3 - C_4)\sin\beta x - (C_3 + C_4)\cos\beta x\}$$

$$= \beta^2 e^{-\beta x}\{-2C_4\cos\beta x + 2C_3\sin\beta x\}$$

$y''_{x=0} = 0$ より. $C_4 = 0$.

$$y''' = -\beta^3 e^{-\beta x}(+2C_3\sin\beta x) + \beta^2 e^{-\beta x}\cdot 2C_3\beta\cos\beta x = -\beta^3 e^{-\beta x}(2C_3)(\sin\beta x - \cos\beta x)$$

$$Q(x) = -EIy''' \quad : \quad Q(0) = -EI\cdot\beta^2\cdot 2C_3\beta = -P$$

$$\therefore C_3 = \frac{P}{2EI\beta^3}$$

$$\therefore y = \frac{P}{2EI\beta^3}e^{-\beta x}\cos\beta x$$

$$y' = \frac{-P}{2EI\beta^2}e^{-\beta x}(\cos\beta x + \sin\beta x)$$

$$y'' = \frac{P}{EI\beta}e^{-\beta x}\sin\beta x$$

$$y''' = \frac{P}{EI}e^{-\beta x}(\cos\beta x - \sin\beta x)$$

$$M(x) = -EIy'' = -\frac{P}{\beta}e^{-\beta x}\sin\beta x$$

$x = 0.785/\beta$ の位置で

$$M_{max} = -\frac{P}{\beta}e^{-0.785}\sin(0.785) = -0.322\frac{P}{\beta}$$

$$Q(x) = -EIy''' = -Pe^{-\beta x}(\cos\beta x - \sin\beta x)$$

杭頭変位 $y_{x=0} = \dfrac{P}{2EI\beta^3}$ ※杭頭固定の2倍.

$Q(x) = 0$ の位置 M が最大.

$\cos\beta x - \sin\beta x$

$= \sqrt{2}\cos(\beta x + \theta)$ $\therefore \beta x + \theta = \frac{\pi}{2}$

$\theta = \tan^{-1}(1)$ $\therefore \theta = \frac{\pi}{4}$ $\beta x = \frac{\pi}{4}$

$x = 0.785/\beta$